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## WHAT EXACTLY IS TIME INFINITY FOR ACOUSTICAL PARAMETERS?

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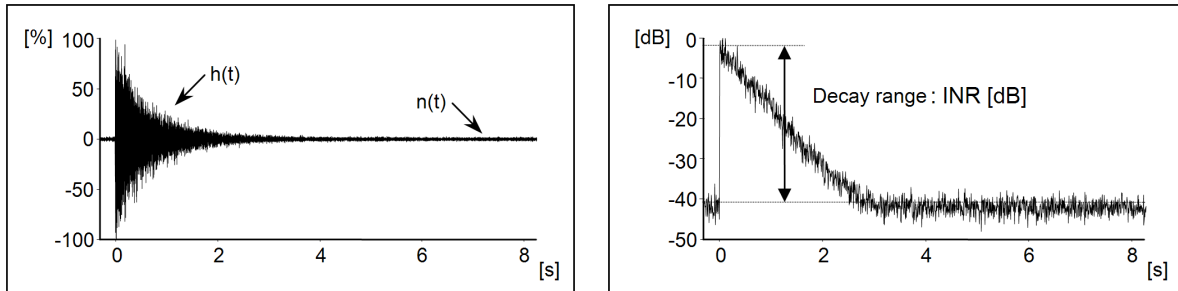
Standards for calculating room acoustical parameters, such as ISO 3382, in many cases do not, or only partially, specify the requirements that a measured impulse response should meet to allow calculation of a certain parameter. For instance, to determine the reverberation time, clear requirements are stated with regard to the exponential shape of the decay curve and the associated decay range, i.e. the  $T_{30}$  can only be calculated if the INR (Impulse response to Noise Ratio)  $> 45$  dB while for  $T_{20}$  the requirement is  $\text{INR} > 35$  dB. To calculate the Strength (G) from an impulse response, ISO 3382 requires a minimum decay range (INR) of 30 dB, and sets the integration time limit to the point where the decay curve has decreased by at least 30 dB. For the Inter Aural Cross Correlation (IACC) an integration time limit in the order of the reverberation time is suggested, irrespective of the noise level. For the Support (ST) the integration time limit for the late energy is set to 1 s. For calculating energy related parameters such as Clarity ( $C_{80}$ ) and Definition ( $D_{50}$ ) no practical value is specified for the late energy integration time limit. It is therefore often left to the user of the standard to find a practical interpretation of the time infinity that appears in the theoretical formulas defining the parameters. Under the most adverse conditions this may lead to variations in the calculated parameter values larger than the JND. The influence of the INR and the integration time limit on the calculated parameter values are investigated. The result is a proposal for practical values of the integration time limits based on the INR and the reverberation time.

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### 1. Introduction

Fig. 1 shows an example of a measured impulse response and its derived Energy Time Curve (ETC). The usability of a measured impulse response (for deriving acoustical parameters) is mainly determined by the available decay range. To this end the  $\text{INR}^1$  (Impulse response to Noise Ratio) is used as a quality parameter. Many room acoustic parameters are derived from the room impulse responses. Examples of such parameters are the reverberation time, which is related to the energy decay rate, the clarity, the definition and the centre time, which are related to early to late energy ratios, and the speech intelligibility, which is related to the energy modulation transfer characteristics of the impulse response. Because for energy parameters no requirements are stated in the stan-

dards with regard to the quality of an impulse response, two of them have been investigated, being the clarity  $C_{80}$  and the centre time  $T_S$ .



**Figure 1.** Measured impulse response  $p(t)$  and energy time curve  $\lg[p^2(t)]$ .  
 $h(t)$ : actual system response,  $n(t)$ : noise, INR: impulse response to noise ratio.

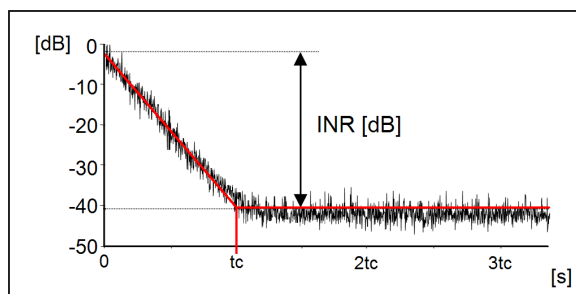
The definitions of these parameters according to ISO 3382-1<sup>2</sup>, are given by Eqs. (1) and (2). The  $C_{80}$ <sup>3</sup> is calculated from the squared impulse response:

$$C_{80} = 10 \lg \left( \frac{\int_0^{80ms} p^2(t) dt}{\int_{80ms}^{\infty} p^2(t) dt} \right) \quad [dB] \quad (1)$$

The parameter  $T_S$ <sup>4</sup> is calculated from the squared impulse response using the following relation:

$$T_S = \frac{\int_0^{\infty} t \cdot p^2(t) dt}{\int_0^{\infty} p^2(t) dt} \cdot 1000 \quad [ms] \quad (2)$$

In theory these formulas hold for infinite measurement times and zero noise, while obviously neither of these conditions can be met in practice. For practical measurements of finite length, one is tempted to equate infinity with the end of the measurement. However, in the presence of noise this results in parameter values that depend on the length of the measurement. For IACC calculations, ISO 3382 suggests using the reverberation time as the integration limit. This is valid as long as the INR of the impulse response is sufficiently large, and has not been artificially increased using noise compensation. Otherwise, it is still possible to include a relatively large section of noise in the calculations, making the results dependent on the noise level. An approach that includes as much as possible of the decaying impulse response while excluding most of the noise, would be to use the crosspoint of decay line and noise level ( $t_{c(ross)}$ ) as the (infinite) integration limit. Hereafter the infinite designation  $\text{inf}$  will be used rather than the symbol  $\infty$ . The influence of the noise and the integration time limit on calculated and measured parameter values related to  $T_{60}$  and a multiple of the crosspoint for infinity (Fig. 2) are investigated.



**Figure 2.** The crosspoint  $t_c$  is defined as the point where the decay line crosses the noise level.

## 2. Infinity calculations

Practical signals from which the acoustical parameters  $C_{80}$  and  $T_S$  are calculated can be modeled as an ideal impulse response  $h(t)$  with added noise  $n(t)$ :

$$p(t) = A \cdot h(t) + B \cdot n(t) \quad (3)$$

The scaling factors  $A$  and  $B$  are used to model the INR (impulse response to noise ratio) with the following relation:

$$INR = 20 \cdot \lg\left(\frac{A}{B}\right) \quad [dB] \quad (4)$$

The ideal impulse response  $h(t)$  is assumed to start at  $t=0$  and has an exponential decay related to the reverberation time  $T_{60}$ :

$$h(t) = r(t) \cdot 10^{\frac{-3t}{T_{60}}} \quad (5)$$

Where  $r(t)$  is a carrier signal with RMS value of 1.

### 2.1 $C_{80}$ calculations (exponential)

The  $C_{80}$  is calculated from the squared impulse response:

$$C_{80} = 10 \lg \left( \frac{\int_0^{80ms} p^2(t) dt}{\int_{80ms}^{t_{inf}} p^2(t) dt} \right) \quad [dB] \quad (6)$$

For practical responses of limited length that include noise, the infinity in the limit of the denominator integral needs further specification.

$$\int_{t_1}^{t_2} p^2(t) dt = \int_{t_1}^{t_2} A^2 \cdot h^2(t) + B^2 \cdot n^2(t) + 2 \cdot A \cdot h(t) \cdot B \cdot n(t) dt \quad (7)$$

Because the signals  $h(t)$  and  $n(t)$  are uncorrelated with average value 0, the integral over their product is 0. Using:

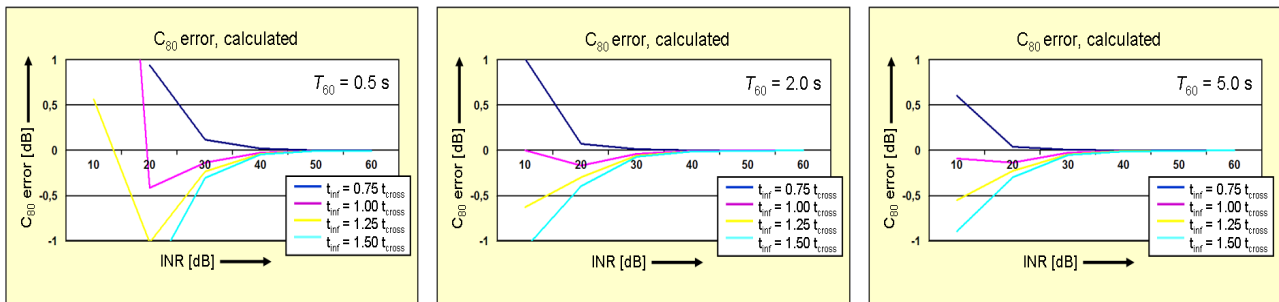
$$K = \frac{T_{60}}{6 \cdot \ln(10)} \quad (8)$$

where  $T_{60}$  is the reverberation time, expressed in seconds, based on the decay rate over a dynamic range of 60 dB, normally starting at 5 dB below the initial level of the decay curve.

We can now write:

$$\int_{t_1}^{t_2} p^2(t) dt = A^2 \cdot K \cdot \left[ e^{-\frac{t_1}{K}} - e^{-\frac{t_2}{K}} \right] + B^2 \cdot (t_2 - t_1) \quad (9)$$

The crosspoint is defined as the point where the decay line crosses the noise level. A plot of  $C_{80}$  as a function of the INR, taking a multiple of the crosspoint time for infinity, shows that the 35 dB stated in ISO 3382 as the minimum required decay range is valid, assuming infinity is taken to be 0.75 to 1.00  $t_{\text{cross}}$  and  $T_{60}$  is at least 0.5 s. With INR values below 35 dB the  $C_{80}$  becomes highly dependent on the integration limit, and quickly drops outside the JND (= 1 dB) limits. In particular, taking the crosspoint itself as the integration limit seems to be a good choice. Fig. 3 shows the influence of the noise and the integration time on calculated  $C_{80}$  error values related to  $T_{60}$  and a multiple of the crosspoint for infinity.



**Figure 3.** The influence of the noise and the integration time limit on calculated  $C_{80}$  error values related to  $T_{60}$  and a multiple of the crosspoint time for infinity.

## 2.2 $T_S$ calculations (exponential)

The parameter  $T_S$  is calculated from the squared impulse response using the following relation:

$$T_S = \frac{\int_0^{t_{\text{inf}}} t \cdot p^2(t) dt}{\int_0^{t_{\text{inf}}} p^2(t) dt} \cdot 1000 \quad [ms] \quad (10)$$

For practical responses of limited length that include noise, the infinity in both limits needs further specification.

$$\int_0^{t_{\text{inf}}} p^2(t) dt = \int_0^{t_{\text{inf}}} A^2 \cdot h^2(t) + B^2 \cdot n^2(t) + 2 \cdot A \cdot h(t) \cdot B \cdot n(t) dt \quad (11)$$

and

$$\int_0^{t_{\text{inf}}} t \cdot p^2(t) dt = \int_0^{t_{\text{inf}}} t \cdot [A^2 \cdot h^2(t) + B^2 \cdot n^2(t) + 2 \cdot A \cdot h(t) \cdot B \cdot n(t)] dt \quad (12)$$

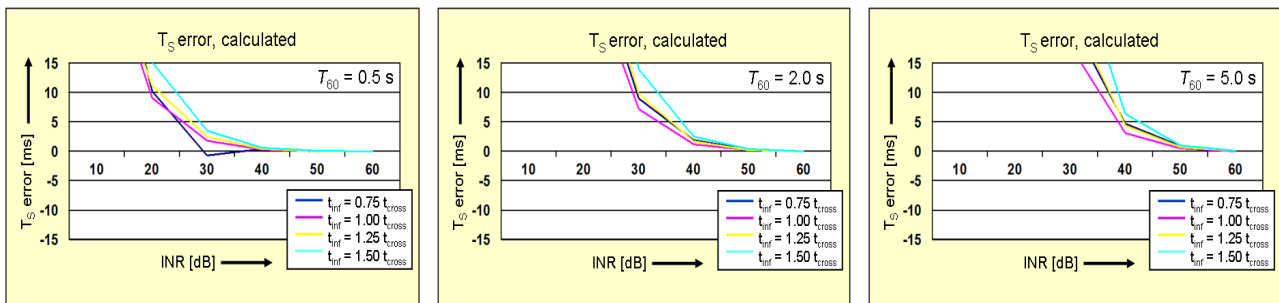
Because the signals  $h(t)$  and  $n(t)$  are uncorrelated with average value 0, the integral over their product is 0. We can write:

$$\int_0^{t_{\text{inf}}} p^2(t) dt = A^2 \cdot K \cdot \left[ 1 - e^{-\frac{t_{\text{inf}}}{K}} \right] + B^2 \cdot t_{\text{inf}} \quad (13)$$

and

$$\int_0^{t_{\text{inf}}} t \cdot p^2(t) dt = A^2 \cdot K^2 + A^2 \cdot K \cdot e^{-\frac{t_{\text{inf}}}{K}} \cdot (t_{\text{inf}} + K) + B^2 \cdot t_{\text{inf}}^2 \quad (14)$$

The plot of  $T_S$  error as a function of the INR where  $t_{\text{inf}}$  is taken to be a multiple of the crosspoint position, shows that in all cases the INR must be higher than 35 dB. Also, with increasing INR the  $T_S$  converges to the value for the theoretical case where no noise is present in the response signal. The best choice (by a small margin) for  $t_{\text{inf}}$  is the crosspoint position. Fig. 4 shows the influence of the noise and the integration time on calculated  $T_S$  error values related to  $T_{60}$  and a multiple of the crosspoint for infinity.



**Figure 4.** The influence of the noise and the integration time limit on calculated  $T_S$  error values related to  $T_{60}$  and a multiple of the crosspoint for infinity.

### 3. Infinity measurements

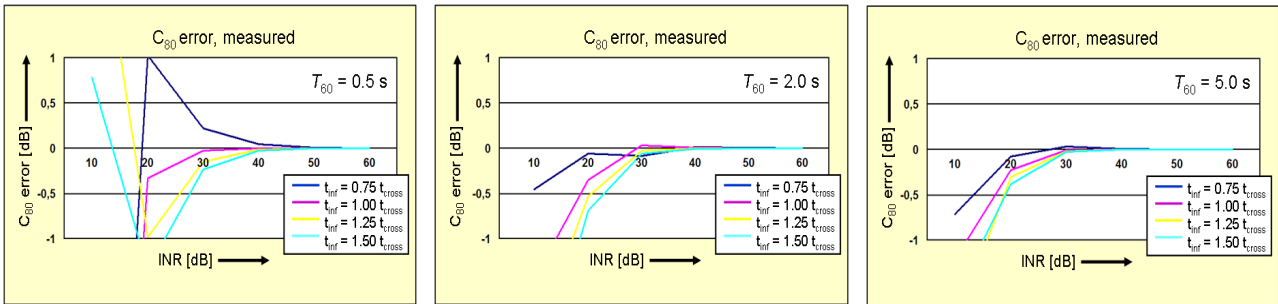
The time infinity is also investigated by analysing 3 measured impulse responses of extremely high quality, each mixed with synthetic noise. Table 1 shows  $C_{80}$  and  $T_S$  for each impulse response, in the 1 kHz octave frequency band, at a measurement length equal to  $T_{60}$  and INR exceeding 80 dB. The addition of synthetic noise resulted in 3 x 6 new impulse responses, with the INR ranging from 10 through 60 dB. From each new impulse response  $C_{80}$  and  $T_S$  were calculated, where  $t_{\text{inf}}$  (Eqs. (6) and (10)) was varied from  $0.75t_{\text{cross}}$  to  $1.50t_{\text{cross}}$ , and where  $t_{\text{cross}}$  is the point where the decay line crosses the noise level (Fig. 2). The starting point of the impulse response is fixed at  $t = 0$ .

**Table 1. Used impulse responses for 1 kHz octave band and measurement length =  $T_{60}$ .**

Room	Vol [ $m^3$ ]	INR [dB]	$T_{60}$ [s]	$C_{80}$ [dB]	$T_S$ [ms]
Reverberant	90	86	5.0	-4.89	353.1
Concert hall	13000	80	2.0	3.13	86.6
Not reverberant	100	97	0.5	9.14	37.0

### 3.1 $C_{80}$ measurements

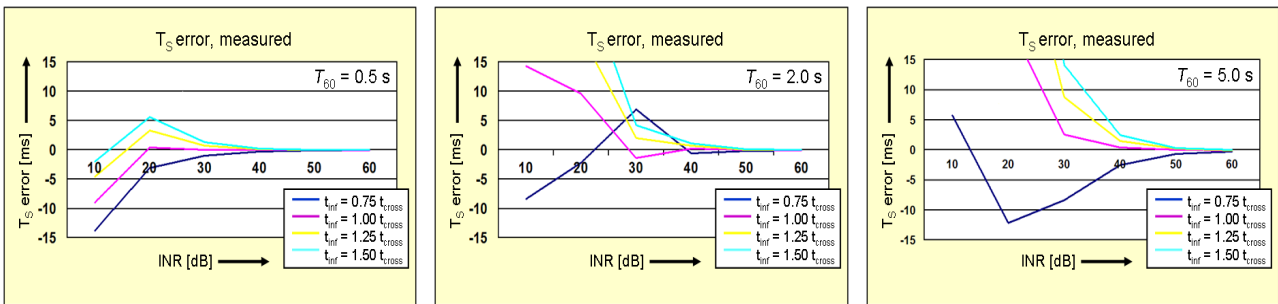
Fig. 5 depicts the clarity  $C_{80}$  measurement error as a function of reverberation time  $T_{60}$ , decay range INR and the chosen infinity, given as a multiple of the crosspoint time value. It shows that in all cases the INR must be higher than 35 dB. Also, with increasing INR the  $C_{80}$  converges to the value for the theoretical case where no noise is present in the response signal. The best choice for  $t_{inf}$  is the crosspoint position.



**Figure 5.** The influence of the noise and the integration time limit on measured  $C_{80}$  error values related to  $T_{60}$  and a multiple of the crosspoint for infinity.

### 3.2 $T_S$ measurements

Fig. 6 shows the centre time  $T_S$  measurement error as a function of reverberation time  $T_{60}$ , decay range INR and the chosen infinity, given as a multiple of the crosspoint time value. It shows that in all cases the INR must be higher than 35 dB. Also, with increasing INR the  $T_S$  converges to the value for the theoretical case where no noise is present in the response signal. The best choice for  $t_{inf}$  is again the crosspoint position.



**Figure 6.** The influence of the noise and the integration time limit on measured  $T_S$  error values related to  $T_{60}$  and a multiple of the crosspoint for infinity.

## 4. Results and discussion

**Table 2. Calculated and measured parameter value errors for  $t_{inf} = t_{cross}$  and INR = 35 dB.**

Reverberation Time $T_{60}$	$t_{inf} = t_{cross}$ and INR = 35 dB			
	$C_{80}$ error (JND = 1 dB)		$T_S$ error (JND = 10 ms*)	
	Calculated	Measured	Calculated	Measured
$T_{60} = 0.5$ s	< 0.10 dB	< 0.05 dB	< 1.0 ms	< 0.2 ms
$T_{60} = 2.0$ s	< 0.02 dB	< 0.02 dB	< 5.0 ms	< 0.2 ms
$T_{60} = 5.0$ s	< 0.02 dB	< 0.01 dB	< 10 ms	< 2.0 ms

**Table 3. Calculated and measured parameter value errors for  $t_{inf} = t_{cross}$  and INR = 45 dB.**

Reverberation Time $T_{60}$	$t_{inf} = t_{cross}$ and INR = 45 dB			
	$C_{80}$ error (JND = 1 dB)		$T_S$ error (JND = 10 ms)	
	Calculated	Measured	Calculated	Measured
$T_{60} = 0.5$ s	< 0.010 dB	< 0.010 dB	< 0.5 ms	< 0.1 ms
$T_{60} = 2.0$ s	< 0.005 dB	< 0.010 dB	< 1.0 ms	< 0.2 ms
$T_{60} = 5.0$ s	< 0.005 dB	< 0.005 dB	< 2.0 ms	< 0.5 ms

## 5. Conclusions

Using the crosspoint of the decay line and the noise level as an optimal approximation of infinite time in energy ratio parameters, the following can be concluded on the impact on noise and measurement time on the room acoustic parameters clarity  $C_{80}$  and centre time  $T_S$ :

- In all cases the INR value must be higher than 35 dB, the same value that is needed to calculate  $T_{20}$ .
- Parameter errors obtained from practical impulse responses tend to be much smaller than calculated errors obtained from comparable theoretical impulse responses based on pure exponential energy decays.
- Using an INR value of 35 dB (the minimum INR for  $T_{20}$  calculations), the measured  $C_{80}$ -error, at least for reverberation times exceeding 0.5 s, is less than 0.05 dB. In this case the  $T_S$ -error is less than 2 ms.
- Using an INR value of 45 dB (the minimum INR for  $T_{30}$  calculations), the measured  $C_{80}$ -error at least for reverberation times exceeding 0.5 s, is less than 0.01 dB. In this case the  $T_S$ -error is less than 0.5 ms.

## 6. Acknowledgements

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